Secondary Markets for Mobile Data: Feasibility and Benefits of Traded Data Plans

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***Abstract*—The growing volume of mobile data traffic has led many Internet service providers (ISPs) to cap their users' monthly data usage, with steep overage fees for exceeding their caps. In this work, we examine a secondary data market in which users can buy and sell leftover data caps from each other. China Mobile Hong Kong recently introduced such a market. While similar to an auction in that users submit bids to buy and sell data, it differs from traditional double auctions in that the ISP serves as the middleman between buyers and sellers. We derive the optimal prices and amount of data that different buyers and sellers are willing to bid in this market and then propose an algorithm for ISPs to match buyers and sellers. We compare the resulting matching for different ISP objectives and derive conditions under which an ISP can obtain higher revenue with the secondary market: while the ISP loses revenue from overage fees, it can assess administration fees and pocket the differences between the buyer and seller prices. Finally, we use one year of usage data from 100 U.S. mobile users to illustrate that the conditions for a revenue increase can hold in practice.**

I. Introduction

Most Internet service providers (ISPs), facing large growth in their network traffic, have attempted to limit excessive data usage by charging users a fixed fee for a maximum amount of data usage in a month, i.e., a monthly data cap [1]. Usage over the cap requires paying steep overage fees, and cannot generally be rolled over into subsequent months [2], [3]. Yet consumers are heterogeneous in the amount of data that they use over a month: some users may use relatively little data, always remaining under their data cap, while others may often purchase additional data as they exceed their data caps [4].

A. Traded Data Plans

The discrepancy between heterogeneous data usage and fixed data caps has been somewhat mitigated by shared data plans [5]–[8]. Such plans allow data caps to be shared across multiple users and devices; thus, heavy users can reduce the probability of exceeding their data caps by sharing a cap with light users, who effectively give away some of their data caps to heavier users. Yet most users only share data plans with their immediate family. If all family members use similar amounts of data, they may still use significantly less or significantly more data than their shared data cap [5].

While most users will not give away their leftover data caps to strangers, they might *sell* their leftover data. Heavy users could then purchase additional data directly from other users, avoiding ISPs' high overage fees. However, ISPs would still need to be involved in this secondary market for data, both to enforce the traded data caps in users' bills (e.g., ensuring that buyers are not charged overage fees for their purchased data), and to help buyers and sellers find each other (e.g., through an exchange platform). China Mobile Hong Kong (CMHK) recently introduced such a secondary market [9]. CMHK's 2cm (2nd exchange market) data exchange platform allows users to submit bids to buy and sell data, with CMHK acting as a middleman both to match buyers and sellers and to ensure that the sellers' trading revenue and buyers' purchased data are reflected on customers' monthly bills.

To the best of our knowledge, no research paper has yet studied traded data plans. Thus, several important research questions remain unanswered: *how do users choose the bids to submit, and how would an ISP match buyers to sellers?* More fundamentally, *why would ISPs offer such data plans?*

At first glance, we would expect ISPs to lose revenue with the secondary market: instead of purchasing overage data from the ISP, users can buy data directly from other users, who may offer lower prices.[1] However, the ISP's status as a middleman between buyers and sellers allows it to extract revenue from buyer-seller transactions. In this work, *we derive the optimal buyer, seller, and ISP behavior. We show that all three parties can benefit from the option of a secondary market, both analytically and with simulations over a one-year dataset of 100 users' monthly usage from a U.S. ISP.*

B. Related Work

Most previously studied data auctions aim to mitigate network congestion. For example, [10] considers a scheme in which users place bids on each transmitted data packet and the ISP admits packets in order of decreasing bids. The authors in [11] consider a similar "transport auction" to distribute uplink capacity among users with delay-tolerant traffic. The work in

[1] Eventually, some users may buy lower data caps from the ISP since they can buy data from other users; conversely, others may buy high data caps and resell them to other users. There may be long-term branding and marketing benefits, rather than just monetary reasons, for an ISP in offering a secondary market. We do not consider these long-term effects in this paper, instead focusing on user and ISP behavior within one month.

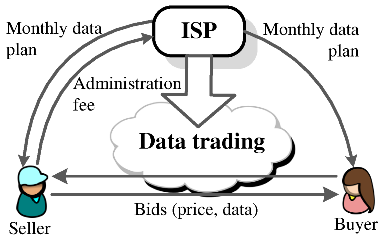


Fig. 1. Buyer-seller interaction in the traded data plan.

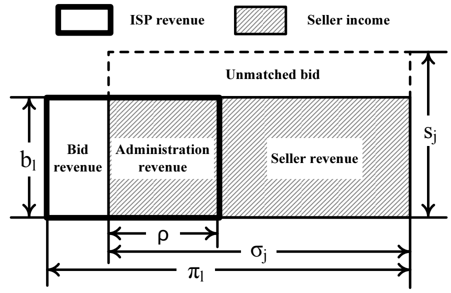


Fig. 2. Buyer-seller matching with their bids and ISP revenue.

[12] allocates different bandwidth to users based on quality of service requirements specified in their bids, while [13] supposes that users send demand functions to an auctioneer who calculates users' prices and their capacity allocation.

Secondary markets can also occur in spectrum auctions, with secondary spectrum holders purchasing temporary spectrum from the primary holders [14]. The spectrum capacity, however, is only held on a temporary basis, introducing different buyer and seller incentives than those for data trading. Moreover, spectrum auctions do not have a third-party middleman, a trait shared by more generic double auction works for electronic commerce and electrical power [15]–[17].

C. Modeling User and ISP Behavior

We suppose that each seller (resp. buyer) can submit a bid to the secondary market consisting of the volume of data he or she wishes to sell (buy) and the unit price he or she is willing to accept (pay) for the data. The ISP then matches buyers and sellers to each other. While the ISP determines the amount of data that users can buy or sell, a buyer always pays her bid price for any data bought, and similarly a seller always receives his bid price (any differences between the amounts paid and received go to the ISP). Thus, users have no incentive to lie about the prices they are willing to accept (sellers) or pay (buyers). Figure 1 shows this buyer-seller interaction.

Choosing optimal bids (Section II): When choosing how much data to bid, users must account for its effect on their usage in the rest of the month, which also depends on their unknown future usage preferences. For instance, buyers may use more data if they can buy data in the secondary market. However, users might not be able to trade their entire bid amount; thus, if they benefit more from trading a very small amount of data rather than an amount near the optimum, they may bid a smaller amount of data. *We show that it is optimal for users to assume they can trade their entire bid and derive the resulting amount of data to bid as a function of the bid price, accounting for its effect on future usage.*

The prices that users bid affect whether their bid can be fully matched: for instance, some buyers may not pay the high price set by a seller. However, users do not know how much of their bid can be matched without knowing the ISP's matching algorithm and other users' bids. The user must therefore guess his or her chance at being matched if he or she bids a certain price. We first examine ISPs' matching policies before giving an algorithm for users to estimate.

Matching buyers and sellers (Section III): The ISP matches users so as to optimize its revenue, including volume-based administration fees and "bid" revenue, or the price difference between buyers who pay high and sellers willing to accept low prices. Since buyers will buy more data in the secondary market due to its low prices compared to ISP overage fees, the ISP can collect substantial administration fees, which can exceed its primary market revenue. *We compare the users matched when the ISP optimizes its different types of revenue and derive conditions under which the ISP can gain revenue compared to the primary market.*

Market dynamics (Section IV): As they participate in more matchings, users form an estimate of the amount of data they can buy or sell at different prices. Some users, however, might not use these estimates to choose their bid prices, e.g., optimistic sellers may always try to sell data at high prices, even if they are unlikely to be matched. *We propose an algorithm for users to adjust their expectations of being matched and change their bid prices accordingly.*

We finally simulate the day-to-day market interactions over a one-year dataset of monthly usage for 100 U.S. ISP customers in Section V. We show that buyers, sellers, and the ISP can all benefit from the secondary market, depending on how much data buyers are willing to purchase. We conclude the paper in Section VI.

II. USER TRADING BEHAVIOR

We consider L buyers who purchase data from other users and J sellers who sell their leftover data. In this section, we discuss how sellers (Section II-A) and buyers (Section II-B) choose their bids to maximize their utilities,[2] and then consider how users choose whether to become a buyer or seller in Section II-C. Since users can choose whether or not to participate in the secondary market, they can benefit from having the option of participating. We now introduce notation and behaviorial considerations common to buyers and sellers.

Since different users can purchase different data caps from their ISPs [2], we denote a buyer l and seller j's data caps before trading as d_l^b and d_j^s respectively. Each buyer and seller has a maximum amount of leftover data, denoted as o_l^b and o_j^s; thus, each user consumes at least $d_l^b - o_l^b$ (buyers) or $d_j^s - o_j^s$ (sellers) amount of data. For instance, users will likely have some predictable usage over a month, e.g., for habitual browsing and checking email. Note that this leftover data must be less than the data cap: $o_l^b \leq d_l^b$ and $o_j^s \leq d_j^s$.

[2]The utility maximization may be performed by a third-party agent working on behalf of buyers and sellers.

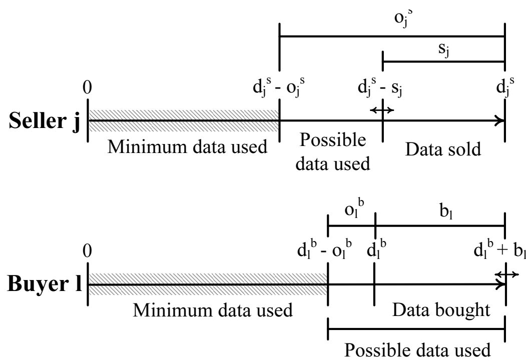


Fig. 3. Relationships between the data caps (d_j^s and d_l^b), leftover data (o_j^s and o_l^b), and data sold/bought (s_j and b_l) for sellers and buyers.

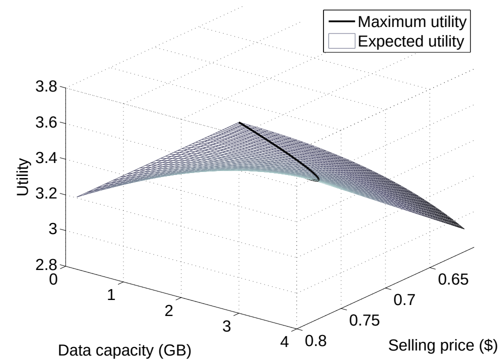


Fig. 4. Illustration of the seller's maximum utility. Price ($/GB) and data parameters are $\alpha_j^s = 0.4$, $\theta_j^s = 1$, $d_j^s = 5$, $o_j^s = 3$, and $\rho = 15$.

We define a buyer l's bid by an amount of data b_l and a price π_l that she is willing to pay. Similarly, each seller j bids a price σ_j for an amount of data s_j. Figure 2 shows how buyers' bids are matched to sellers' bids and how the ISP receives revenue in the secondary market. In this case, the buyer purchases her entire bid. The seller's income is split between the *administration revenue* paid to the ISP and the revenue kept by the seller. The ISP pockets *bid revenue* from the difference between buyer and seller prices. The bid prices are lower bounded by an administration fee ρ per unit data sold that the ISP imposes on the sellers, as in CMHK's traded data plan [9]. Sellers will not accept a buyer l's price $\pi_l < \rho$, since π_l does not cover the administration fee and the seller loses money. The prices are upper-bounded by the ISP's overage fee p per unit data: buyers prefer to buy data from the ISP at price p rather than accept seller j's price if $\sigma_j > p$.

Absent the cost or revenue from trading data, users gain utility from consuming data. We use α-fair utility functions to model the usage utility from consuming c amount of data:

$$V(c) = \frac{\theta c^{1-\alpha}}{1-\alpha}, \tag{1}$$

where θ is a positive constant representing the scale of the usage utility and we take $\alpha \in [0, 1)$. We use $\left(\theta_j^s, \alpha_j^s\right)$ to denote the parameters for seller j and $\left(\theta_l^b, \alpha_l^b\right)$ for buyer l.

A. Sellers' Optimal Bids

Since sellers can submit bids before the end of the month, they do not exactly know their future monthly usage. Thus, we suppose that each seller j's realized usage c_j^s for the month is a random variable with distribution f. This distribution depends not only on the amount of data sold s_j, but also on the user's maximum leftover data o_j^s and data cap before trading d_j^s.

Figure 3 shows that the seller consumes at least $d_j^s - o_j^s$ amount of data, i.e., his minimum usage, and at most $d_j^s - s_j$ amount of data, i.e., the data cap after selling data.[3] The jth seller's expected usage utility from selling s_j amount of data is then $\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s$. The seller's revenue term equals $(\sigma_j - \rho)s_j$ (Figure 2), so the expected utility of the jth seller when selling s_j data is given by:

$$E(U_j^s \mid s_j) = \int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s + (\sigma_j - \rho)s_j. \tag{2}$$

We note that (2) is always increasing in the price σ_j. *Thus, sellers always bid high prices, subject to their ability to be matched to buyers (Section IV).* Given σ_j and the distribution f, the seller chooses $s_j^\star(\sigma_j) \in [0, o_j^s]$ so as to maximize the utility (2). Though it is possible that the seller will not be able to sell all of his data, it is still optimal for the seller to bid the utility-maximizing $s_j^\star$: If $E(U_j^s \mid s_j)$ is concave, then $E(U_j^s \mid s_j)$ is increasing in s_j for $s_j \in [0, s_j^\star]$. Thus, the seller always bids the maximum amount of data up to the optimum amount.[4] We now show some example distributions for which $E(U_j^s \mid s_j)$ is concave.

Example Distributions. Some sellers may only use the minimum data (i.e., f is a delta distribution centered at $d_j^s - o_j^s$.) In this case, $E(U_j^s \mid s_j)$ is linear in s_j and the seller bids $s_j^\star = o_j^s$ amount of data.

Other sellers may use up their entire data caps in the month, i.e., f is the delta distribution centered at $d_j^s - s_j$. In this case, the utility function in (2) can be written as:

$$E_\delta(U_j^s \mid s_j) = V_j^s(d_j^s - s_j) + (\sigma_j - \rho)s_j. \tag{3}$$

Thus, we compute the optimal bid as $s_j^\star = \max\left\{0, \min\left\{o_j^s, d_j^s - \left((\sigma_j - \rho)/\theta_j^s\right)^{-1/\alpha_j^s}\right\}\right\}$.

In most cases, the seller's usage will fall somewhere between these two extremes. We thus follow [5] in supposing that it follows a uniform distribution $f(c_j^s) = (o_j^s - s_j)^{-1}$ between $d_j^s - o_j^s$ and $d_j^s - s_j$. In this case, we first show that $E(U_j^s \mid s_j)$ is a concave function:

Proposition 1: The utility function of the jth seller $E(U_j^s \mid s_j)$ in (2) is concave in s_j if $f(c_j^s)$ is a uniform

[3] Users constrain their usage below $d_j^s - s_j$ so as to avoid having to buy more data later in the month.

[4] We show in Section III-A that bidding an amount $s_j > s_j^\star$ does not increase the seller's probability of matching exactly $s_j^\star$ amount of data.

distribution. Then, the optimal bid $s_j^\star$ satisfies

$$(o_j^s - s_j^\star)(\sigma_j - \rho) = V_j^s(d_j^s - s_j^\star) - \int_{d_j^s - o_j^s}^{d_j^s - s_j^\star} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s. \quad (4)$$

Figure 4 shows the value of the utility corresponding to all the possible σ_j and s_j; (4) is satisfied along the black curve. We now observe that $s_j^\star$ is decreasing in σ_j:

Corollary 1: The optimal amount sold $s_j^\star(\sigma_j)$ for each seller j increases as σ_j increases if $E(U_j^s \mid s_j)$ is concave.

To solve for $s_j^\star$ using (4), we use the nonlinear Perron-Frobenius theory [18] (cf. Appendix A):

Algorithm 1 Sellers' Utility Maximization

Initialize $\mathbf{s}(0) \in (\mathbf{0}, \mathbf{o}^s)$.

1) The jth seller updates the data caps to be sold:

$$s_j(k+1) = o_j^s - \frac{1}{\sigma_j - \rho} V_j^s(d_j^s - s_j(k)) + \frac{1}{\sigma_j - \rho} \int_{d_j^s - o_j^s}^{d_j^s - s_j(k)} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s.$$

2) Normalize $s_j(k+1)$:

$$s_j(k+1) \leftarrow \min \left\{ s_j(k+1), d_j^s - \left(\frac{\theta_j^s \left(d_j^s + \alpha_j^s o_j^s \right)}{2 d_j^s (\sigma - \rho)} \right)^{\frac{1}{\alpha_j^s}} \right\}.$$

Lemma 1: Algorithm 1 converges geometrically fast to the fixed point $s_j^\star$ in (4) from any initial point $s_j(0)$ if $s_j^\star \leq d_j^s - \left(\theta_j^s (1 + \alpha_j^s o_j^s / d_j^s) / (2(\sigma_j - \rho))\right)^{1/\alpha_j^s}$.

Since the right-hand side of Lemma 1's condition is decreasing in the utility scaling factor θ_j^s, we expect it to be satisfied for relatively low values of θ_j^s. For such θ_j^s, the user will have relatively low utility from usage, as we would expect from a seller. We formalize this intuition in Section II-C.

B. Buyers' Optimal Bids

Like the sellers, buyers do not exactly know their future usage. Thus, we take the buyer's monthly usage c_l^b to be a random variable with distribution $f(c_l^b)$ between the minimum usage $d_l^b - o_l^b$ and data cap after trading $d_l^b + b_l$ (Figure 3). Hence, the expected data usage utility of the lth buyer purchasing b_l amount of data is given by $\int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b$. Each buyer l's cost of purchasing b_l amount of data is $b_l \pi_l$, so the expected utility of the lth buyer is

$$E(U_l^b \mid b_l) = \int_{d_l^b - o_l^b}^{d_l^b + b_l} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b - b_l \pi_l. \quad (5)$$

Since (5) is decreasing in π_l, *buyers wish to bid at lower prices, subject to their ability to be matched to sellers (Section IV).* As with the seller, the buyer will always bid her utility-maximizing $b_l^\star$ if $E(U_l^b \mid b_l)$ is concave.

Example Distributions. As in Section II-A, some buyers will use only their minimum usage $d_l^b - o_l^b$; these buyers will therefore not purchase any data in the market. Other buyers will use up their entire data cap, i.e., their distribution f will be the delta distribution centered at $d_l^b + b_l$. The utility function under this delta distribution is given by

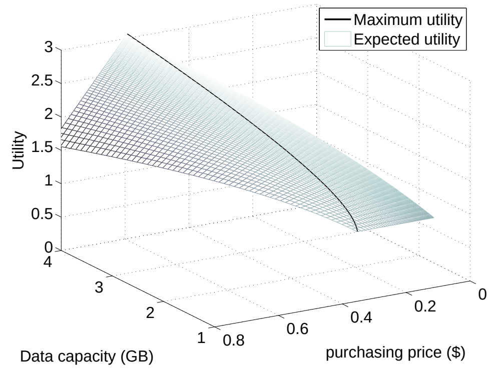


Fig. 5. Illustration of the buyer's maximum utility. Price (\$/GB) and valuations are $\alpha_l^b = 0.4$, $\theta_l^b = 1$, $d_l^b = 1$, and $o_l^b = 0.2$.

$$E_\delta(U_l^b \mid b_l) = V_l^b(d_l^b + b_l) - \pi_l b_l, \quad (6)$$

yielding the optimal data bid $b_l^\star(\pi_l) = \left(\pi_l / \theta_l^b\right)^{-1/\alpha_l^b}$.

In most cases, however, the buyer's usage will lie between the two extremes. We thus consider the case where f is the uniform distribution $f(c_l^b) = 1/(o_l^b + b_l)$. We first show that the utility in (5) is concave:

Proposition 2: The utility function of the lth buyer $E(U_l^b \mid b_l)$ in (5) is concave in b_l if $f(c_l^b)$ is a uniform distribution. Then, the optimal bid $b_l^\star$ satisfies:

$$(o_l^b + b_l^\star)\pi_l = V_l^b(d_l^b + b_l^\star) - \int_{d_l^b - o_l^b}^{d_l^b + b_l^\star} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b. \quad (7)$$

Figure 5 shows the value of the utility corresponding to all the possible π_l and b_l, where (7) is satisfied along the black curve. We also note that $b_l^\star$ is a decreasing function of the price:

Corollary 2: The optimal bid $b_l^\star(\sigma_j)$ for each buyer l decreases as σ_j increases if $E(U_l^b \mid b_l)$ is concave.

We again use the Perron-Frobenius theory to solve for $b_l^\star$:

Algorithm 2 Buyers' Utility Maximization

Initialize $\mathbf{b}(0) \in \mathbb{R}_+^L$.

1) The lth buyer updates the amount of data to be purchased:

$$b_l(k+1) = \frac{1}{\pi_l} V_l^b(d_l^b + b_l(k)) - \frac{1}{\pi_l} \int_{d_l^b - o_l^b}^{d_l^b + b_l(k)} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b - o_l^b.$$

2) Normalize $b_l(k+1)$:

$$b_l(k+1) \leftarrow \min \left\{ b_l(k+1), \left(\frac{\theta_l^b (d_l^b + \alpha_l^b o_l^b)}{2 d_l^b \pi_l} \right)^{\frac{1}{\alpha_l^b}} - d_l^b \right\}.$$

Lemma 2: Algorithm 2 converges geometrically fast to the fixed point $b_l^\star$ in (7) from any initial point $b_l(0)$ if $b_l^\star \leq \left(\theta_l^b (1 + \alpha_l^b o_l^b / d_l^b) / (2\pi_l)\right)^{1/\alpha_l^b} - d_l^b$.

We thus observe that the algorithm converges for buyers with high utility scaling factors θ_l^b. We show in the next section that buyers will likely satisfy this condition.

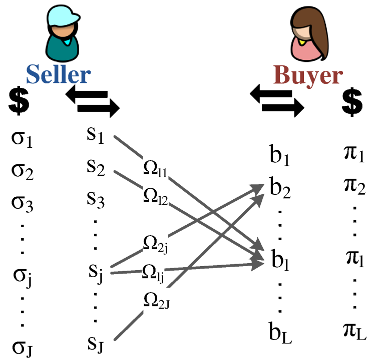


Fig. 6. Illustration of the process of matching sellers and buyers.

C. Selling or Buying Data

Users choose to become a buyer or seller based on the utility they can achieve from buying or selling data. Thus, if

$$E(U_j^s \mid s_j^\star(p)) \geq E(U_l^b \mid b_l^\star(\rho)), \tag{8}$$

the user becomes a seller: The user's maximum utility from selling data (assuming all data is sold at the maximum price) must be higher than the maximum utility from purchasing data (assuming all data is bought at the minimum price).[5] If (8) is reversed, the user buys data.

To illustrate this decision, we suppose that the user's usage follows the delta distribution, as in Sections II-A and II-B's examples. We then derive the following condition on users' utility scaling factor θ in in the usage utility function (1):

Corollary 3: The user sells data when the scale θ satisfies $\theta \leq \hat{\theta}$ and buys data when $\theta \geq \hat{\theta}$, where

$$\hat{\theta} = \left(\left(\frac{1-\alpha}{\alpha} \right) \left(\frac{(p-\rho)d_j^s - \rho d_l^b}{\rho^{\frac{\alpha-1}{\alpha}} - (p-\rho)^{\frac{\alpha-1}{\alpha}}} \right) \right)^{\alpha}. \tag{9}$$

Thus, users with high utility scaling θ become buyers, while those with low θ become sellers.

III. ISP Trading Policies

The ISP will match buyers and sellers so as to optimize its revenue, subject to constraints imposed by the buyer and seller bids (Section III-A). We analyze the optimal matching in Section III-B before considering whether the resulting revenue exceeds that of the primary data market in Section III-C.

A. ISP Optimization

The ISP will often encounter sellers' and buyers' bids that are not exactly aligned: for instance, if a seller offers more data than any single buyer is willing to purchase. To help match such bids, we suppose that the ISP can match multiple buyers to multiple sellers. Since the ISP acts as a middleman between all buyers and all sellers, this flexibility is transparent to users. All required accounting can be done internally by the ISP.

We denote the matching between buyers and sellers with a matrix $\mathbf{\Omega} = [\Omega_{lj}]_{l,j=1}^{L,J} \geq 0$. Each (l,j) entry of $\mathbf{\Omega}$ represents the percentage of the lth buyer's demand (i.e., amount of data bid) b_l that is satisfied by the jth seller's data supply s_j; thus, $\Omega_{lj} b_l$ represents the amount of data that buyer l purchases from seller j.[6] Note that the ISP can take any bids from users (e.g., $s_j = s_j^\star(\sigma_j)$ and $b_l = b_l^\star(\pi_l)$) in the matching optimization. Figure 6 shows a schematic of the buyer and seller interaction.

1) Matching Constraints: The ISP's matching is primarily constrained by the buyer and seller bids. Buyer l's bid of a price π_l and amount of data b_l constrains the ISP matching in two ways: first, the buyer will buy at most b_l amount of data, leading to the feasible set

$$\mathcal{B} = \left\{ \mathbf{\Omega} \,\middle|\, \sum_{j=1}^{J} \Omega_{lj} \leq 1, \quad l = 1, \ldots, L \right\}. \tag{10}$$

We thus suppose that the buyer will accept matchings in which her bid is only partially matched (Section II).

Second, the buyer's price π_l gives an upper bound to the average purchase price of her data. We assume that the buyer will pay this bid price π_l for all data purchased; the resulting amount paid, $\pi_l \sum_j \Omega_{lj} b_l$, must be at least as much as the data cost specified by sellers' bid prices (i.e., a cost $\sigma_j \Omega_{lj} b_l$ for each seller j). Mathematically, we have the feasible set

$$\Pi = \left\{ \mathbf{\Omega} \,\middle|\, \sum_{j=1}^{J} \Omega_{lj}\sigma_j \leq \pi_l \sum_{j=1}^{J} \Omega_{lj}, \; l = 1, \ldots, L \right\}. \tag{11}$$

If the total amount paid by the buyer exceeds the data cost, the ISP keeps the excess as part of its bid revenue.

Similarly, seller j's bid of a price σ_j and amount of data s_j implies that he will sell at most s_j amount of data:

$$\mathcal{S} = \left\{ \mathbf{\Omega} \,\middle|\, \sum_{l=1}^{L} \Omega_{lj} b_l \leq s_j, \quad j = 1, \ldots, J \right\}. \tag{12}$$

In return, the total money paid by all buyers for seller j's data $\sum_l \Omega_{lj} b_l \pi_l$ must be at least the cost of the data $\sigma_j \sum_l \Omega_{lj} b_l$:

$$\Sigma = \left\{ \mathbf{\Omega} \,\middle|\, \pi_l \sum_{l=1}^{L} \Omega_{lj} b_l \geq \sigma_j \sum_{l=1}^{L} \Omega_{lj} b_l, \; j = 1, \ldots, J \right\}. \tag{13}$$

Thus, the ISP must choose $\mathbf{\Omega} \in \mathcal{B} \cap \Pi \cap \mathcal{S} \cap \Sigma$, which can be written as a set of linear constraints as in (10)-(13).

Intuitively, if sellers bid sufficiently low and buyers sufficiently high prices, they can be matched to at least one other user. We derive these price thresholds using (11) and (13):

Proposition 3 (Price feasibility): If seller j sells data to at least one buyer ($\sum_l \Omega_{lj} b_l > 0$), then his selling price σ_j is lower than at least one buyer's purchasing price: $\sigma_j \leq \max_l \pi_l$.

[5]Here we assume the user can always sell or buy all the bid data. More generally, the user could estimate the maximum amount of data he or she could sell or buy at a given price using past experience (Section IV); the amount of data sold is the minimum of this quantity and the optimal bid amount. Users buy (sell) data if the resulting utility is higher for selling (buying) data at the prices maximizing these utilities.

[6]Equivalently, we can define Ω_{lj} as the percentage of user j's supply sold to buyer l, which has the effect of transposing the matching constraints.

Analogously, if buyer l purchases data from at least one seller (i.e., $\sum_j \Omega_{lj} > 0$), then her purchasing price is higher than at least one seller's selling price, i.e., $\pi_l \geq \min_j \sigma_j$.

2) ISP Objective: The ISP's objective in choosing a matching $\mathbf{\Omega}$ is to maximize its revenue from the secondary market. We identify two sources of ISP revenue: "administration revenue" and "bid revenue" (Figure 2).

The ISP's revenue from the administration fee is proportional to the volume of data traded, i.e., $\rho \sum_{l,j} \Omega_{lj} b_l$. To calculate the bid revenue, we sum the gaps between each buyer's payment and each seller's income: $\sum_l \left(\pi_l \sum_j \Omega_{lj} b_l - \sum_j \sigma_j \Omega_{lj} b_l\right)$. From (11), this gap is always positive. The ISP thus maximizes its revenue by solving the linear program

$$\begin{aligned} \underset{\mathbf{\Omega}}{\text{maximize}} \quad & \omega\rho \sum_{j=1}^{J} \sum_{l=1}^{L} \Omega_{lj} b_l \\ & + (1-\omega) \sum_{l=1}^{L} \sum_{j=1}^{J} (\Omega_{lj} b_l \pi_l - \Omega_{lj} b_l \sigma_j) \\ \text{subject to} \quad & \mathbf{\Omega} \in \mathcal{B} \cap \mathcal{S} \cap \Pi \cap \Sigma; \mathbf{\Omega} \geq \mathbf{0}. \end{aligned} \tag{14}$$

The parameter ω trades off between administration revenue and bid revenue; its effect is our next subject of discussion. We use $\mathbf{\Omega}^\star$ to denote the optimal solution to (14).

We note that, if a seller bids $\tilde{s}_j > s_j^\star$, it does not improve his chance of having $\sum_l \Omega_{lj}^\star b_l^\star = s_j^\star$ at the optimality of (14), but he may have $s_j^\star < \sum_l \Omega_{lj}^\star b_l^\star = \tilde{s}_j$, yielding suboptimal utility for the seller. Similarly, buyers do not bid more than their optimal amount $b_l^\star$.

B. Matching Buyers and Sellers

The ISP can maximize its total revenue by choosing $\omega = 0.5$, i.e., weighting the bid and administration revenue equally. However, while both types of revenue generally increase with the amount of data traded, changing ω can lead to different matching solutions. The ISP can thus incorporate other considerations into its matching objective.

Taking $\omega < 0.5$, i.e., preferentially weighting the bid revenue, is equivalent to reducing the administration fee ρ. For instance, if we assume that the ISP incurs a constant marginal cost of supporting $\sum_{l,j} \Omega_{lj} b_l$ amount of traffic on its network, it can subtract this cost from ρ; the ISP thus maximizes its profit instead of its revenue.

When the ISP preferentially weights its bid revenue, it attempts to match buyers with high prices to sellers with low prices, increasing the difference in the amount paid by buyers and sellers. In contrast, when maximizing its administration revenue, the ISP wishes to maximize the total amount of data traded. Thus, for higher ω (i.e., preferential weight to administration revenue) the ISP might match a seller to buyers with both higher and lower prices; buyers' prices π_l would then average out to equal the seller's price σ_j, and the seller would be able to trade more data than if he had only been matched to buyers with higher π_l. Indeed, we can derive a specific condition on ω for which such matchings occur:

Proposition 4 (Matching feasibility): If $\pi_l < \sigma_j$ and

$$\omega < \frac{\max_j \sigma_j - \min_l \pi_l}{\rho + (\max_j \sigma_j - \min_l \pi_l)}, \tag{15}$$

then the ISP will not match buyer l to seller j.

We note that if $\omega = 1$, then ω never satisfies (15), and low price buyers can be matched to sellers with higher prices. The amount matched of a user's bid can thus depend on not only others' bids, but also the ISP's matching objective.

Figure 7 illustrates the effect of varying ω with the matching results for five users. When the ISP preferentially weights bid revenue, i.e., ω is small, the seller with the lowest price (Seller 1) and the buyer with highest price (Buyer 3) are the only users matched. However, as ω increases, more users are matched; in fact, for $\omega > 0.44$, buyers 1 and 4 both purchase data, even though their bid prices are lower than all the sellers' bid prices. Furthermore, as ω increases, the administration revenue $\rho \sum_j \sum_l \Omega_{lj} b_l$ increases, but the bid revenue $\sum_l \sum_j (\Omega_{lj} b_l \pi_l - \Omega_{lj} b_l \sigma_j)$ decreases.

Even for $\omega = 1$, some buyers and sellers may not be matched to any users. We can in fact derive price thresholds for buyers and sellers above (resp. below) which all buyers (sellers) can trade some data, and below (above) which no buyer (seller) trades any data:

Proposition 5 (Price competition): Suppose that sellers are sorted with price ascending ($\sigma_{j+1} \geq \sigma_j$) and buyers sorted with price descending ($\pi_{l+1} \leq \pi_l$). Then $\mathbf{\Omega}^\star$ is a block matrix with all the non-zero entries in the northwest corner:

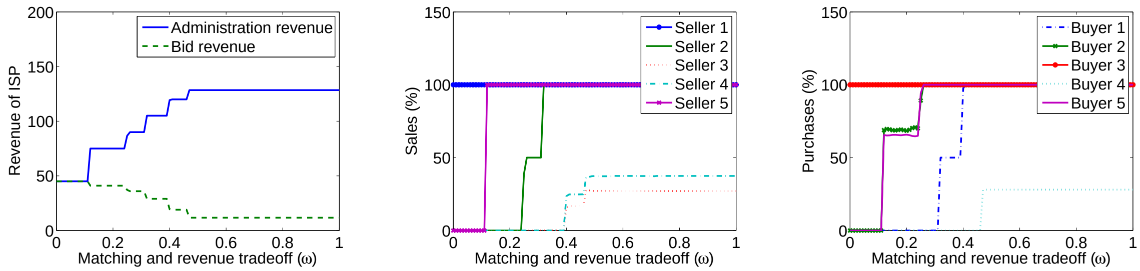
1) If the mth buyer is not matched with any seller ($\sum_j \Omega_{mj} = 0$), then all buyers $l > m$ (i.e., whose bid prices are lower than that of buyer m) are also unmatched: all the entries below an all-zero row in $\mathbf{\Omega}^\star$ are zero.
2) If the nth seller is not matched with any buyer ($\sum_l \Omega_{ln} = 0$), then all sellers $j > n$ (i.e., whose bid prices are higher than that of seller n) are also unmatched: all the entries to the right of an all-zero column in $\mathbf{\Omega}^\star$ are zero.

Thus, buyers and sellers compete with each other on the basis of price. Buyers paying higher prices and sellers accepting lower prices are rewarded by the ability to trade some data.

C. Comparison to the Primary Market

In the absence of a secondary market, buyers would be forced to buy overage data at price p from the ISP instead of from other users. Thus, in the secondary market, buyers purchase more data due to lower prices. Since the ISP receives administration revenue in proportion to the amount of data sold in the secondary market, its revenue can be larger than the revenue earned in the primary market. Table I compares users' utilities and ISP revenue in both markets.

Figure 8 illustrates ISP and user behavior in the primary and secondary markets for the simplified case of one buyer and one seller. In the primary market, buyers purchase data from the ISP at the maximum price p. The lth buyer thus maximizes



(a) ISP Revenue. (b) Matched percentage of sellers' data caps. (c) Matched percentage of buyers' data caps.

Fig. 7. ISP revenue and user matching with $\rho = 15$, $p = 60$, $\mathbf{s} = (3,2,3,2,2)^\top$, $\boldsymbol{\sigma} = (35,45,48,48,42)^\top$, $\mathbf{b} = (2,1,3,2,2)^\top$ and $\boldsymbol{\pi} = (35,45,50,35,40)^\top$. Seller 1 and Buyer 3 offer the lowest and highest prices respectively, and can always trade all their data. Users with the highest selling price (Seller 3 and 4) and the lowest purchasing price (Buyer 4) can trade data when ω is sufficiently large (Proposition 4).

TABLE I

COMPARISON OF USER UTILITY AND ISP REVENUE IN THE PRIMARY MARKET AND THE SECONDARY MARKET

	Primary market	**Secondary market**
Buyer l	$E(U_l^b \,\|\, b_l^\star(p)) = \int_{d_l^b - o_l^b}^{d_l^b + b_l^\star(p)} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b - \pi_l b_l^\star(p)$	$E(U_l^b \,\|\, b_l^\star(\pi_l)) = \int_{d_l^b - o_l^b}^{d_l^b + b_l^\star(\pi_l)} V_l^b(c_l^b) f(c_l^b) \mathrm{d}c_l^b - \pi_l b_l^\star(\pi_l)$
Seller j	$E(U_j^s \,\|\, s_j^\star = 0) = \int_{d_j^s - o_j^s}^{d_j^s} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s$	$E(U_j^s \,\|\, s_j^\star(\sigma_j)) = \int_{d_j^s - o_j^s}^{d_j^s - s_j^\star(\sigma_j)} V_j^s(c_j^s) f(c_j^s) \mathrm{d}c_j^s + (\sigma_j - \rho) s_j^\star(\sigma_j)$
ISP	$p \sum_{l=1}^{L} b_l^\star(p)$	$\rho \sum_{j=1}^{J} s_j^\star(\sigma_j) + \left(\sum_{l=1}^{L} \pi_l b_l^\star(\pi_l) - \sum_{j=1}^{J} \sigma_j s_j^\star(\sigma_j) \right)$

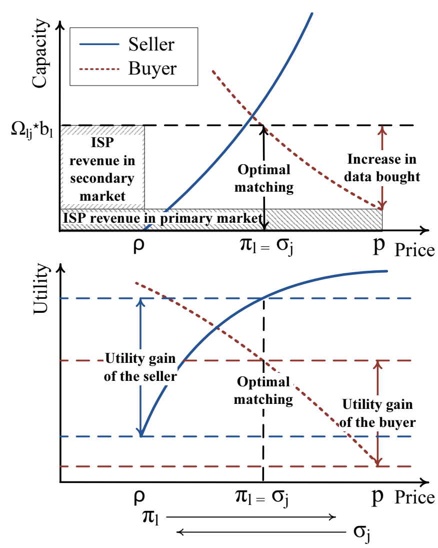


Fig. 8. As illustrated here for one seller and one buyer, users always increase their utilities and the ISP can earn more revenue in the secondary market.

her utility by purchasing $b_l^\star(p)$ data from the ISP. Hence, the revenue of the ISP in the primary market is $p \sum_{l=1}^{L} b_l^\star(p)$. Sellers do not participate in the primary market, which is equivalent to taking $\sigma_j = \rho$ in the secondary market: at this price, the seller does not earn any revenue from selling data and loses utility if he sells data; therefore $s_j^\star = 0$.

In Figure 8, the sellers and buyers are matched when their prices align ($\sigma_j = \pi_l$). Thus, in the secondary market, the lth buyer purchases $b_l^\star(\pi_l)$ amount of data, where $\pi_l < p$. Since $b_l^\star(\cdot)$ monotonically decreases with respect to the price (Corollary 2), $b_l^\star(p) < b_l^\star(\pi_l)$ and the buyer purchases more data in the secondary than in the primary market. However, this increase in data may not allow the ISP to recover the revenue lost from the primary market:

Proposition 6 (Revenue benefit): A necessary condition for the ISP to earn more revenue in the secondary market than in the primary market is

$$\frac{p}{\rho} \leq \min_{l,\ldots,L} \frac{b_l^\star(\rho)}{b_l^\star(p)}. \qquad (16)$$

For instance, if the buyers' future usage distributions are delta distributions as in Section II-B, $b_l^\star(\pi_l) = (\pi_l/\theta_l^b)^{-1/\alpha_l^b}$. Thus, $b_l^\star(\rho)/b_l^\star(p) = (p/\rho)^{1/\alpha_l^b} > p/\rho$, and the ISP can earn more revenue in the secondary market.

IV. DYNAMIC DATA TRADING

In practice, buyers and sellers can submit new bids at any time, so the ISP must run many matchings over a month. Note that the number of matchings can vary from month to month depending on how frequently users submit bids. Moreover, buyers and sellers can learn from the results of each matching: for instance, if the jth seller is not matched to any buyers, he can lower the price of his next bid to attract more buyers.

Optimistic buyers (resp. sellers) might choose their initial prices $\pi_l(0) = \rho$ or $\sigma_j(0) = p$, though buyers would likely have to raise and sellers lower their prices before they could be matched. Risk-averse sellers and buyers, on the other hand, would respectively choose close to the minimum and maximum prices to ensure that they will be matched.

Other users would leverage their past experience. For instance, the buyer estimates the expected amount of data $g_l^b(\pi_l)$ that she could buy as a function of the bid price π_l and then computes her optimal data bid $b_l^\star(\pi_l)$. The buyer would then expect to be able to purchase $\min\{g_l^b(\pi_l), b_l^\star(\pi_l)\}$ amount of data, yielding a utility $E\left(U_l^b \,|\, \min\{g_l^b(\pi_l), b_l^\star(\pi_l)\}\right)$. The buyer then chooses her initial price $\pi_l(0) \geq \rho$ so as to maximize this utility. The function $g_l^b(\pi_l)$ can depend on the time of the month at which the bid is submitted, e.g., there are likely to be more buyers at the end of the month as users run out of their data caps, and can be updated at the end of each month. Sellers can choose their prices $\sigma_j(0)$ analogously.

In each iteration k, we assume that the buyers and sellers increase and decrease their prices by $\epsilon_l^b(k)$ and $\epsilon_j^s(k)$ respectively, subject to the constraints that $\pi_l(k), \sigma_j(k) \in [\rho, p]$. Users can set ϵ based on their transaction history, e.g., as described above for the initial price, and/or their risk preferences: a larger ϵ changes the price more, increasing the likelihood of being matched but lowering their utilities.

We incorporate these initial prices and price adjustment in the following dynamics, which are formalized in Algorithm 3:

1) Initially, users choose $\pi_l(0)$ and $\sigma_j(0)$ as above. They then calculate the optimal amounts of data to bid $b_l(0) = b_l^\star(\pi_l)$ and $s_j(0) = s_j^\star(\sigma_j)$ as in Algorithms 1 and 2.
2) Upon receiving bids from at least one seller and one buyer, the ISP runs the matching optimization (14).
3) Users respond to the matching result from the ISP. If only a portion of the bid is matched, each buyer l increases her price by an amount ϵ_l^b and each seller l decreases her price by an amount ϵ_j^s. They then recompute the amount of data to bid with this new price and submit the new bids to the ISP.

Algorithm 3 Data Trading Dynamics

At $k = 0$, each seller and buyer initializes $\sigma_j(0)$, $s_j^\star(\sigma_j(0))$ and $\pi_l(0)$, $b_l^\star(\pi_l(0))$ respectively.

while $L(k) > 0$ **and** $J(k) > 0$ **do**

1) Upon receiving bids $\big(b_l(k), \pi_l(k)\big)$ and $\big(s_j(k), \sigma_j(k)\big)$ from $L(k)$ buyers and $J(k)$ sellers, the ISP updates the constraint sets $\mathcal{B} \equiv \mathcal{B}(k)$, $\Pi \equiv \Pi(k)$, $\mathcal{S} \equiv \mathcal{S}(k)$ and $\Sigma \equiv \Sigma(k)$.

2) The ISP computes $\mathbf{\Omega}(k+1)$ by solving (14) with $L(k)$, $J(k)$, $s_j(k)$, $\sigma_j(k)$, $b_l(k)$, $\pi_l(k)$ as L, J, s_j, σ_j, b_l, π_l.

3) Each seller j updates the bid price and amount of data:

if $\sum_l \Omega_{lj}(k+1) b_l(k) < s_j(k)$ **then**
$d_j^s(k+1) = d_j^s(k) - \sum_l \Omega_{lj}(k+1) b_l(k)$,
$o_j^s(k+1) = o_j^s(k) - \sum_l \Omega_{lj}(k+1) b_l(k)$,
$\sigma_j(k+1) = \max\{\sigma_j(k) - \epsilon_j^s(k), \rho\}$,
Run Algorithm 1 to obtain $s_j(k+1)$.
end if
$J(k+1) = J(k)$.
if $\sum_l \Omega_{lj}(k+1) b_l(k) = s_j(k)$ **then**
Transaction is successful: $J(k+1) = J(k+1) - 1$.
end if

4) Each buyer l updates the bid price and amount of data:

if $\sum_j b_l(k) \Omega_{lj}(k+1) < b_l(k)$ **then**
$d_l^b(k+1) = d_l^b(k) + \sum_j b_l(k) \Omega_{lj}(k+1)$,
$o_l^b(k+1) = o_l^b(k) + \sum_j b_l(k) \Omega_{lj}(k+1)$,
$\pi_l(k+1) = \min\{\pi_l(k) + \epsilon_l^b(k), p\}$,
Run Algorithm 2 to obtain $b_l(k+1)$.
end if
$L(k+1) = L(k)$.
if $\sum_j b_l(k) \Omega_{lj}(k+1) = b_l(k)$ **then**
Transaction is successful: $L(k+1) = L(k+1) - 1$.
end if

5) New sellers and buyers submit bids to the ISP.
A new seller submits bid: $J(k+1) = J(k+1) + 1$,
A new buyer submits bid: $L(k+1) = L(k+1) + 1$.
6) $k = k + 1$.

end while

V. Numerical Evaluation

A. Trading Dynamics

We now analyze the data trading dynamics in Algorithm 3 with a five-user example. Figure 9 shows the fraction of their total bids that each seller and buyer trades in each iteration. In Figures 9(a) and 9(b), $\omega = 0$ (i.e., the ISP optimizes its bid revenue), while in Figures 9(c) and 9(d), $\omega = 1$ (optimizing the administration revenue). The sellers and buyers are ordered respectively in increasing and decreasing order of price.

As shown in Figures 9(b) and 9(d), the matching optimization always matches the sellers with lower prices first; the sellers finish selling their bids in increasing order of their prices. Conversely, as shown in Figures 9(a) and 9(c), the matching optimization always matches the buyers with higher prices first, and the buyers finish purchasing their bids in decreasing order of their prices. Thus, buyers with higher price bids and sellers with lower price bids are more likely to be matched. Moreover, the users in Figures 9(c) and 9(d) ($\omega = 1$) are all matched one iteration earlier than those in Figures 9(a) and 9(b) ($\omega = 0$): the ISP matches more users when optimizing administration revenue rather than bid revenue.

In Figure 10, we suppose that new users enter the market at the third time slot. One new seller submits a 2GB bid with a price higher than the other sellers' highest price; at the same time, one new buyer submits a 2GB bid, with a price lower than the other buyers' lowest price. These prices reflect the fact that new participants do not have the experience to realistically estimate the amount of data they can buy or sell at different prices. However, by adjusting their prices the users adapt quickly: the new seller and buyer finish their trading within three time slots.

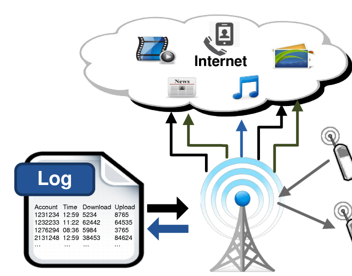
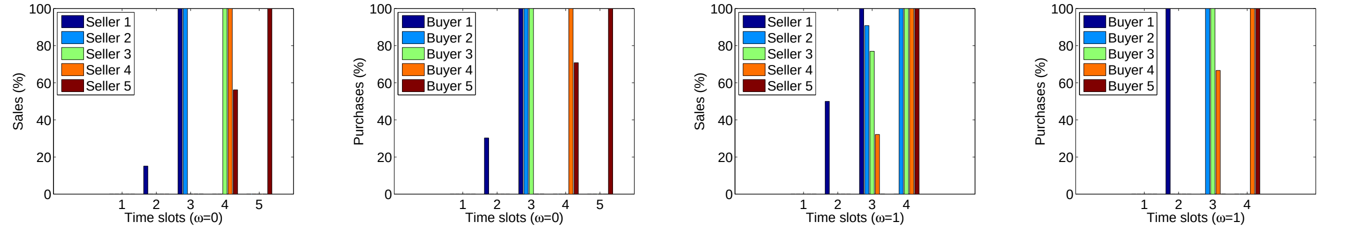


Fig. 11. Mobile usage measurement via in-network RADIUS records.

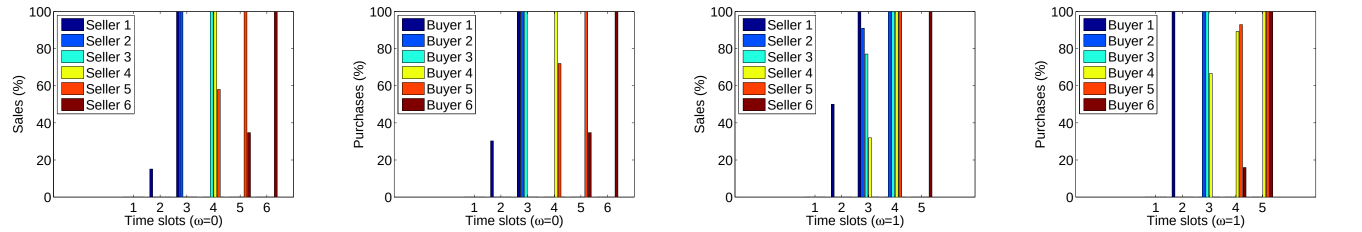
B. Experiments with User Data

We finally simulate Algorithm 3 on real user usage. Our data comes from 100 mobile users of a U.S. ISP from January to December 2013. We measured their usage at a session level via in-network RADIUS records (Figure 11) and use it together with monthly data plans to compute their optimal bids. We classify the users as sellers and buyers using (9). Each user is assumed to have a uniform usage distribution of future usage. Figure 12(a) shows the distribution of buyer and seller bids over all twelve months; we see that sellers' bids are generally



(a) (b) (c) (d)

Fig. 9. Illustration of the data trading framework in Algorithm 3 with five sellers and five buyers. The parameter setting is as follows: $\rho = 15$, $\epsilon = 5$, $p = 60$, $\mathbf{s}(0) = (2,2,2,2,2)^\top$, $\boldsymbol{\sigma}(0) = (52,54,56,58,60)^\top$, $\mathbf{b}(0) = (1,1,2,3,3)^\top$ and $\boldsymbol{\pi}(0) = (42,39,36,33,30)^\top$. We plot the percentage of each buyer/seller's total amount of data bid that has been successfully matched at each iteration. In (a) and (b), $\omega = 0$; while in (c) and (d), $\omega = 1$.



(a) (b) (c) (d)

Fig. 10. Matching in successive timeslots (Algorithm 3) with parameters as in Figure 9. One new buyer and one new seller join the data market at the third time slot. In (a) and (b), $\omega = 0$; while in (c) and (d), $\omega = 1$.

much smaller than buyers', as some buyers bid an enormous amount of data (e.g., users accustomed to regular HD video streaming). However, there are fewer buyers than sellers. Over each month, the users keep trading as described in Algorithm 3 until either sellers' or buyers' bids are all met.

We calculate the total bid and administration revenue for each month and compare this revenue to that of the primary market in Figure 12. In most months, the ISP's administration revenue alone is larger than the revenue from the primary data market due to a large increase in buyers' data purchased. In a few months, e.g., June to September, the primary market yields more revenue: the sellers do not bid enough data to completely satisfy buyers' demand. We can also observe from Figure 12(b) that the gap between the total revenue and administration revenue (i.e., the bid revenue) in the secondary market is slightly larger when $\omega = 0$ than when $\omega = 1$ as in Figure 12(c): at $\omega = 0$, the ISP explicitly maximizes its bid revenue.

Figure 13 shows the total utilities, the utilities of the buyers and the utilities of the sellers in the primary market and secondary market. As we would expect, the utilities of the sellers and buyers in the secondary market are always larger than those in the primary market. The amount of increase, however, varies from month to month.

VI. Conclusion

Since traded data plans have only recently been introduced, the seller, buyer and ISP behaviors are still unknown. We first derive the optimal prices and amount of data that the sellers and the buyers bid to participate in the secondary market, taking into account uncertainty in users' usage and thus the amount of data caps that they need. We then give a condition under which a user will choose to buy or sell data.

The ISP matches buyers and sellers by solving a linear program that maximizes ISP revenue subject to users' bid constraints. We contrast the optimal matchings when bid or administration revenue is emphasized and derive a necessary condition under which the ISP gains revenue in the secondary compared to the primary market. Since the ISP runs this matching many times over the month, we examine how users adjust their bids over time to increase their chances of being matched. Finally, we simulate these dynamics over one year of usage data from a U.S. ISP, demonstrating that the ISP, buyers, and sellers can all benefit from the secondary market.

Much research remains to be done on traded data plans, not only in developing more detailed models of user and ISP behavior, but also in examining how such data plans work in practice (e.g., in CMHK's deployment). Our work takes an initial step towards modeling traded data plans and understanding their benefits for users and ISPs.

Appendix

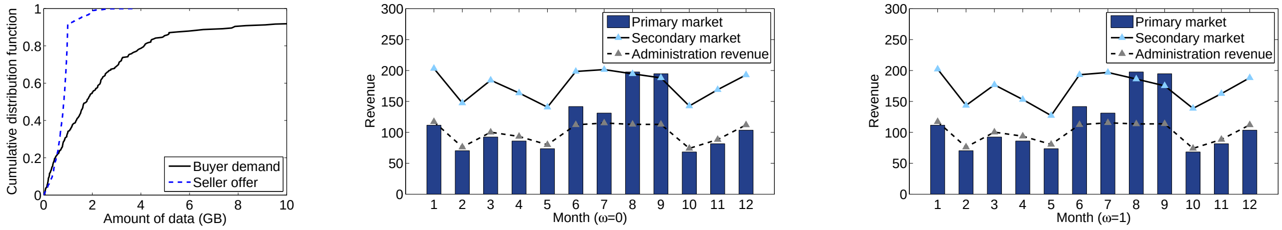
A. *Nonlinear Perron-Frobenius Theory [18]*

Let $\|\cdot\|$ be a monotone norm on $\mathbb{R}^L$. For a concave mapping $f : \mathbb{R}^L_+ \rightarrow \mathbb{R}^L_+$ with $f(\mathbf{z}) > 0$ for $\mathbf{z} \geq \mathbf{0}$, the following statements hold. The conditional eigenvalue problem $f(\mathbf{z}) = \lambda\mathbf{z}$, $\lambda \in \mathbb{R}$, $\mathbf{z} \geq \mathbf{0}$, $\|\mathbf{z}\| = 1$ has a unique solution $(\lambda^*, \mathbf{z}^*)$, where $\lambda^* > 0$, $\mathbf{z}^* > \mathbf{0}$. Furthermore, $\lim_{k\rightarrow\infty} \tilde{f}(\mathbf{z}(k))$ converges geometrically fast to $\mathbf{z}^*$, where $\tilde{f}(\mathbf{z}) = f(\mathbf{z})/\|f(\mathbf{z})\|$.

B. *Proof of Proposition 1*

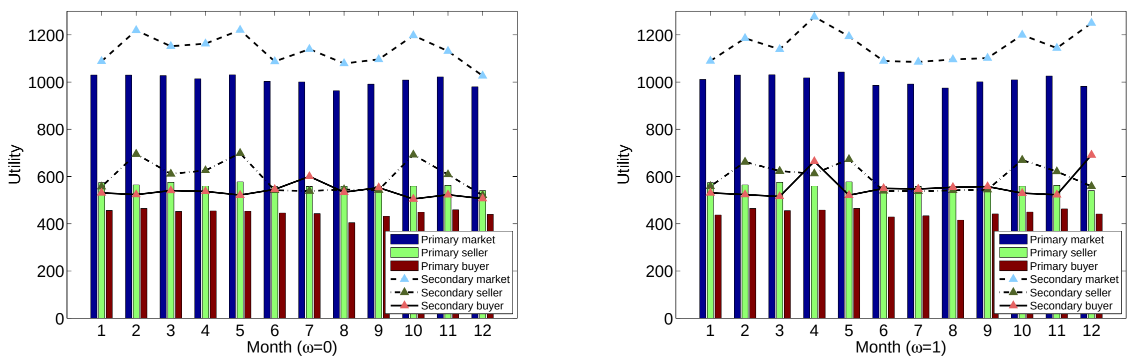
Proof: Taking the second-order derivative of $E(U^s_j \,|\, s_j)$ in (2) with respect to s_j, we have:

$$\mathrm{d}^2 E(U^s_j \,|\, s_j)/\mathrm{d}s_j^2 = \frac{(d^s_j - s_j)^{2-\alpha^s_j}}{(1-\alpha^s_j)(2-\alpha^s_j)(o^s_j - s_j)^3}\Psi(o^s_j),$$



(a) CDF of user bids. (b) Comparison of the revenue ($\omega = 0$). (c) Comparison of the revenue ($\omega = 1$).

Fig. 12. The ISP usually but not always gains revenue in the secondary compared to the primary market ($\rho = 2$, $p = 4$, $\alpha_j^s = \alpha_l^b = 0.6$, $\theta_j^s = 3.3$, $\theta_l^b = 9$).



(a) Comparison of the utility ($\omega = 0$). (b) Comparison of the utility ($\omega = 1$).

Fig. 13. Buyers and sellers always increase the utility in the secondary market (parameters as in Figure 12).

where

$$\Psi(o_j^s) = \frac{2(d_j^s - s_j)^2}{(d_j^s - s_j)^2} - \frac{2(2-\alpha_j^s)(o_j^s - s_j)(d_j^s - s_j)}{(d_j^s - s_j)^2} + \frac{(1-\alpha_j^s)(2-\alpha_j^s)(o_j^s - s_j)^2}{(d_j^s - s_j)^2} - 2\left(\frac{d_j^s - o_j^s}{d_j^s - s_j}\right)^{2-\alpha_j^s}.$$

Next, we show that $\Psi(o_j^s)$ decreases with o_j^s by taking the first-order derivative of $\Psi(o_j^s)$, given by:

$$\begin{aligned}\mathrm{d}\Psi(o_j^s)/\mathrm{d}o_j^s &= \frac{2(2-\alpha_j^s)}{(d_j^s - s_j)^2}\Big(-(d_j^s - s_j) + (1-\alpha_j^s)(o_j^s - s_j) \\ &\quad + (d_j^s - o_j^s)^{1-\alpha_j^s}(d_j^s - s_j)^{\alpha_j^s}\Big) \\ &= \frac{2(2-\alpha_j^s)}{(d_j^s - s_j)^2}\Big(-\alpha_j^s(d_j^s - s_j) - (1-\alpha_j^s)(d_j^s - o_j^s) \\ &\quad + (d_j^s - o_j^s)^{1-\alpha_j^s}(d_j^s - s_j)^{\alpha_j^s}\Big) \le 0,\end{aligned}$$

where the inequality holds due to the inequality of arithmetic-geometric means that $\alpha_j^s(d_j^s - s_j) + (1-\alpha_j^s)(d_j^s - o_j^s) \ge (d_j^s - o_j^s)^{1-\alpha_j^s}(d_j^s - s_j)^{\alpha_j^s}$ for all $\alpha_j^s \in (0,1)$. Since $o_j^s \in [s_j, d_j^s]$, we have

$$\Psi(o_j^s) \le \Psi(o_j^s = s_j) = 0,$$

which also means $\mathrm{d}^2 E(U_j^s \mid s_j)/\mathrm{d}s_j^2 \le 0$. Thus, $E(U_j^s \mid s_j)$ is concave. ∎

C. Proof of Corollary 1

Proof: Consider two prices for seller j, σ_j^1 and σ_j^2, with $\sigma_j^1 < \sigma_j^2$. Then from (4), the optimal amounts sold $s_j^\star$ satisfy

$$\frac{d}{ds_j}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s\right)\Bigg|_{s_j^\star(\sigma_j^i)} = \rho - \sigma_j^i.$$

for $i = 1, 2$. Since $\rho - \sigma_j^1 > \rho - \sigma_j^2$, we have

$$\begin{aligned}&\frac{d}{ds_j}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s\right)\Bigg|_{s_j^\star(\sigma_j^1)} \\ &> \frac{d}{ds_j}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s\right)\Bigg|_{s_j^\star(\sigma_j^2)}.\end{aligned}$$

Since $\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s$ is a concave function by Proposition 1, its first derivative is a decreasing function of s_j. Thus, $s_j^\star(\sigma_j^2) > s_j^\star(\sigma_j^1)$ as desired. ∎

D. Proof of Lemma 1

Proof: We first prove below that the self-mapping function at Step 1 of Algorithm 1 is concave when $s_j \le d_j^s - \left(\frac{1+\alpha_j^s o_j^s/d_j^s}{2(\sigma-\rho)}\right)^{1/\alpha_j^s}$. From (4), we have the following self-mapping function:

$$\begin{aligned}s_j &= g(s_j) \\ &= o_j^s + \frac{1}{\sigma_j - \rho}\left(\int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s - V_j^s(d_j^s - s_j)\right),\end{aligned}$$

i.e., the self-mapping function at Step 1 of Algorithm 1. Hence, $g(s_j)$ is a concave self-mapping function if the following function $h(s_j)$ is concave:

$$h(s_j) = \int_{d_j^s - o_j^s}^{d_j^s - s_j} V_j^s(c_j^s) f(c_j^s)\mathrm{d}c_j^s - V_j^s(d_j^s - s_j).$$

Taking the second-order derivative of $h(s_j)$ with respect to s_j, we have:

$$h''(s_j) = 2(o_j^s - s_j)^{-2}\int_{d_j^s-o_j^s}^{d_j^s-s_j} V_j^s(c_j^s)f(c_j^s)\mathrm{d}c_j^s \\ -2(o_j^s - s_j)^{-2}V_j^s(d_j^s - s_j) + (o_j^s - s_j)^{-1}V_j^{s\prime}(d_j^s - s_j) \\ -V_j^{s\prime\prime}(d_j^s - s_j). \tag{17}$$

Combining (7) with (17), we can obtain:

$$\begin{aligned} h''(s_j) &= -2(o_j^s - s_j)^{-1}(\sigma_j - \rho) \\ &\quad +(o_j^s - s_j)^{-1}V_j^{s\prime}(d_j^s - s_j) - V''(d_j^s - s_j) \\ &= \frac{(o_j^s - s_j)^{-1}}{(d_j^s - s_j)^{\alpha_j^s}}\Bigg(- 2(\sigma_j - \rho)(d_j^s - s_j)^{\alpha_j^s} \\ &\quad +\theta_j^s + \theta_j^s\alpha_j^s\frac{o_j^s - s_j}{d_j^s - s_j}\Bigg). \end{aligned}$$

Due to that $s_j \leq d_j^s - \left(\frac{\theta_j^s(1+\alpha_j^s o_j^s/d_j^s)}{2(\sigma-\rho)}\right)^{1/\alpha_j^s}$, we have

$$\begin{aligned} &(\sigma - \rho)(d_j^s - s_j)^{\alpha_j^s} \geq \frac{\theta_j^s}{2}\left(1 + \alpha_j^s\frac{o_j^s}{d_j^s}\right)^{1/\alpha_j^s} \\ \Rightarrow \quad &(\sigma - \rho)(d_j^s - s_j)^{\alpha_j^s} \geq \frac{\theta_j^s}{2}\left(1 + \alpha_j^s\frac{o_j^s - s_j}{d_j^s - s_j}\right)^{1/\alpha_j^s}, \end{aligned}$$

which implies that $h''(s_j) \leq 0$. Therefore, $h(s_j)$ is concave so that $g(s_j)$ is a concave self-mapping. Furthermore, the normalization at Step 2 of Algorithm 1 is a monotone norm constraint of s_j. Then, the nonlinear Perron-Frobenius theory (cf. Appendix A) can be leveraged for the algorithm design. ■

E. Proof of Proposition 2

Proof: Taking the second-order derivative of $E(U_l^b \,|\, b_l)$ in (5) with respect to b_l, we have:

$$\mathrm{d}E^2(U_l^b \,|\, b_l)/\mathrm{d}b_l^2 = \frac{(d_l^b + b_l)^{2-\alpha_l^b}}{(1-\alpha_l^b)(2-\alpha_l^b)(o_l^b + b_l)}\Psi(o_l^b),$$

where

$$\begin{aligned} \Psi(o_l^b) = &\frac{2(d_l^b + b_l)^2}{(d_l^b + b_l)^2} - \frac{2(2-\alpha_l^b)(o_l^b + b_l)(d_l^b + b_l)}{(d_l^b + b_l)^2} \\ &+\frac{(1-\alpha_l^b)(2-\alpha_l^b)(o_l^b + b_l)^2}{(d_l^b + b_l)^2} - 2\left(\frac{d_l^b - o_l^b}{d_l^b + b_l}\right)^{2-\alpha_l^b}. \end{aligned}$$

Next, we show that $\Psi(o_l^b)$ decreases with o_l^b by taking the first-order derivative of $\Psi(o_l^b)$, given by:

$$\begin{aligned} \mathrm{d}\Psi(o_l^b)/\mathrm{d}o_l^b &= \frac{2(2-\alpha_l^b)}{(d_l^b + b_l)^2}\Big(- (d_l^b + b_l) + (1-\alpha_l^b)(o_l^b + b_l) \\ &\quad +(d_l^b - o_l^b)^{1-\alpha_l^b}(d_l^b + b_l)^{\alpha_l^b}\Big) \\ &= \frac{2(2-\alpha_l^b)}{(d_l^b + b_l)^2}\Big(- (1-\alpha_l^b)(d_l^b - o_l^b) - \alpha_l^b(d_l^b + b_l) \\ &\quad +(d_l^b - o_l^b)^{1-\alpha_l^b}(d_l^b + b_l)^{\alpha_l^b}\Big), \end{aligned}$$

where the inequality holds due to the inequality of arithmetic-geometric means that $(1-\alpha_l^b)(d_l^b - o_l^b) + \alpha_l^b(d_l^b + b_l) \geq (d_l^b - o_l^b)^{1-\alpha_l^b}(d_l^b + b_l)^{\alpha_l^b}$ for all $\alpha_l^b \in (0,1)$. Since $o_l^b \in [0, d_j^s]$, we have

$$\Psi(o_l^b) \leq \Psi(o_l^b = 0) \leq \Psi(o_l^b = -b_l) = 0,$$

which also means $\mathrm{d}E^2(U_l^b \,|\, b_l)/\mathrm{d}b_l^2 \leq 0$. Thus, $E(U_l^b \,|\, b_l)$ is concave. ■

F. Proof of Corollary 2

Proof: Consider two prices for buyer l, π_l^1 and π_l^2, with $\pi_l^1 < \pi_l^2$. Then from (4), the optimal amounts sold $b_l^\star$ satisfy

$$\frac{d}{db_l}\left(\int_{d_l^b-o_l^b}^{d_l^b+b_l} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b\right)\Bigg|_{b_l^\star(\pi_l^i)} = \pi_l^i.$$

for $i = 1, 2$. Since $\pi_l^1 < \pi_l^2$, we have

$$\begin{aligned} &\frac{d}{db_l}\left(\int_{d_l^b-o_l^b}^{d_l^b+b_l} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b\right)\Bigg|_{b_l^\star(\pi_l^1)} \\ &< \frac{d}{db_l}\left(\int_{d_l^b-o_l^b}^{d_l^b+b_l} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b\right)\Bigg|_{b_l^\star(\pi_l^2)}. \end{aligned}$$

Since $\int_{d_l^b-o_l^b}^{d_l^b+b_l} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b$ is a concave function by Proposition 1, its first derivative is a decreasing function of b_l. Thus, $b_l^\star\left(\pi_l^1\right) > b_l^\star\left(\pi_l^2\right)$ as desired. ■

G. Proof of Lemma 2

Proof: Similar to the proof in Appendix D, we first prove below that the self-mapping function at Step in of Algorithm 2 is concave when $b_l \leq \left(\frac{1+\alpha_l^b o_l^b/d_l^b}{2\pi_l}\right)^{1/\alpha_l^b}$. From (7), we have the following self-mapping function:

$$\begin{aligned} b_l &= g(b_l) \\ &= \frac{1}{\pi_l}\left(V_l^b(d_l^b + b_l) - \int_{d_l^b-o_l^b}^{d_l^b+b_l} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b\right) - o_l^b, \end{aligned}$$

i.e., the self-mapping function at Step of Algorithm 2. Hence, $g(b_l)$ is a concave self-mapping function if the following function $h(b_l)$ is concave:

$$h(b_l) = V_l^b(d_l^b + b_l) - \int_{d_l^b-o_l^b}^{d_l^b+b_l} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b.$$

Taking the second-order derivative of $h(b_l)$ with respect to b_l, we have:

$$h''(b_l) = -2(o_l^b + b_l)^{-2}\int_{d_l^b-o_l^b}^{d_l^b+b_l} V_l^b(c_l^b)f(c_l^b)\mathrm{d}c_l^b \\ +2(o_l^b + b_l)^{-2}V_l^b(d_l^b + b_l) - (o_l^b + b_l)^{-1}V_l^{b\prime}(d_l^b + b_l) \\ +V_l^{b\prime\prime}(d_l^b + b_l). \tag{18}$$

Combing (7) with (18), we can obtain:

$$\begin{aligned} h''(b_l) &= 2(o_l^b + b_l)^{-1}\pi_l - (o_l^b + b_l)^{-1}V_l^{b\prime}(d_l^b + b_l) \\ &\quad +V_l^{b\prime\prime}(d_l^b + b_l) \\ &= \frac{(o_l^b + b_l)^{-1}}{(d_l^b + b_l)^{\alpha_l^b}}\Bigg(2\pi_l(d_l^b + b_l)^{\alpha_l^b} \\ &\quad -\theta_l^b - \theta_l^b\alpha_l^b\frac{o_l^b + b_l}{d_l^b + b_l}\Bigg). \end{aligned}$$

Due to that $b_l \leq \left(\frac{\theta_l^b(1+\alpha_l^b o_l^b/d_l^b)}{2\pi_l}\right)^{1/\alpha_l^b} - d_l^b$, we have

$$2\pi_l(d_l^b + b_l)^{\alpha_l^b} \leq \theta_l^b\left(1+\alpha_l^b\frac{o_l^b}{d_l^b}\right)$$
$$\Rightarrow \quad 2\pi_l(d_l^b + b_l)^{\alpha_l^b} \leq \theta_l^b\left(1+\alpha_l^b\frac{o_l^b+b_l}{d_l^b+b_l}\right),$$

which implies that $h''(b_l) \leq 0$. Therefore, $h(b_l)$ is concave so that $g(b_l)$ is a concave self-mapping. Furthermore, the normalization at Step 2 of Algorithm 2 is a monotone norm constraint of b_l. Then, the fixed-point algorithm converges to the unique optimal solution by leveraging the nonlinear Perron-Frobenius theory (cf. Appendix A). ■

H. Proof of Corollary 3

Proof: Considering the special case of delta distribution for the usage utility, the optimality conditions in (4) and (7) can be rewritten respectively as:

$$s_j^\star = d_j^s - \left(\frac{1}{\theta}(\sigma_j - \rho)\right)^{-\frac{1}{\alpha}},$$

and

$$b_l^\star = \left(\frac{\pi_l}{\theta}\right)^{-\frac{1}{\alpha}} - d_l^s.$$

Then, we obtain the maximum utilities for the seller and the buyer, given respectively by

$$E_\delta(U_j^s \,|\, s_j^\star(\sigma_j)) = \frac{\alpha}{1-\alpha}(\sigma_j-\rho)^{1-\frac{1}{\alpha}}\theta^{\frac{1}{\alpha}} + (\sigma_j - \rho)d_j^s,$$

and

$$E_\delta(U_l^b \,|\, b_l^\star(\pi_l)) = \frac{\alpha}{1-\alpha}\pi_l^{1-\frac{1}{\alpha}}\theta^{\frac{1}{\alpha}} - \pi_l d_l^b.$$

Then, by substituting $E_\delta(U_j^s \,|\, s_j^\star(p))$ and $E_\delta(U_l^b \,|\, b_l^\star(\rho))$ back into (8), we can obtain (9). ■

I. Proof of Proposition 3

Proof: Due to $\sum_{j=1}^J \Omega_{lj}b_l \geq 0$ and $\sum_{l=1}^L \Omega_{lj}b_l \geq 0$, the inequality constraints in (11) and (13) can be rewritten respectively, as:

$$\xi_1\sigma_1 + \xi_2\sigma_2 + \cdots + \xi_J\sigma_J \leq \pi_l,$$

where $\xi_j = \Omega_{lj}b_l / \left(\sum_{j=1}^J \Omega_{lj}b_l\right)$ and we have $\xi_1 + \xi_2 + \ldots, \xi_J = 1$, and,

$$\eta_1\pi_1 + \eta_2\pi_2 + \cdots + \eta_L\pi_L \geq \sigma_j,$$

where $\eta_l = \Omega_{lj}b_l / \left(\sum_{l=1}^L \Omega_{lj}b_l\right)$ and we have $\eta_1 + \eta_2 + \cdots + \eta_L = 1$. In other words, π_l should be higher than at least one nonnegative linear combination of all the selling prices $\sigma_1, \ldots, \sigma_J$, and σ_j should be lower than at least one nonnegative linear combination of all the purchasing prices $\pi_l, \ldots, \pi_L$. Since we also have $\min_{\mathbf{1}^\top\boldsymbol{\xi}=1,\boldsymbol{\xi}\geq\mathbf{0}}\{\xi_1\sigma_1 + \xi_2\sigma_2 + \cdots + \xi_J\sigma_J\} = \min_{j=1,\ldots,J}\{\sigma_j\}$ and $\max_{\mathbf{1}^\top\boldsymbol{\eta}=1,\boldsymbol{\eta}\geq\mathbf{0}}\{\eta_1\pi_1 + \eta_2\pi_2 + \cdots + \eta_L\pi_L\} = \max_{l=1,\ldots,L}\{\pi_l\}$, this completes the proof. ■

J. Proof of Proposition 4

Proof: If $\exists l, j$ such that $\Omega_{lj}^\star > 0$ and $\sigma_j > \pi_l$, we have $Z_{lj}^\star = 0$ and (21) can then be rewritten as:

$$\omega = \frac{(x_l^\star + y_j^\star) + (\mu_l^\star + \nu_j^\star + 1)(\sigma_j - \pi_l)}{\rho + (\sigma_j - \pi_l)},$$

which leads to the following inequality:

$$\omega \geq \frac{\sigma_j - \pi_l}{\rho + (\sigma_j - \pi_l)}, \tag{19}$$

since the dual variables $x_l^\star$, $y_j^\star$, $\mu_l^\star$ and $\nu_j^\star$ are nonnegative. Then, (15) is sufficient for (19). ■

K. Proof of Proposition 5

Proof: We form the Lagrangian for (14) by introducing the dual variables $\mathbf{Z} \in \mathbb{R}_+^{L\times J}$, $\mathbf{x} \in \mathbb{R}_+^L$, $\mathbf{y} \in \mathbb{R}_+^J$, $\boldsymbol{\mu} \in \mathbb{R}_+^L$ and $\boldsymbol{\nu} \in \mathbb{R}_+^J$ respectively for the constraints $\Omega_{lj} \geq 0$, $l = 1,\ldots,L$, $j = 1,\ldots,J$, $\sum_{j=1}^J \Omega_{lj}b_l \leq b_l$, $l = 1,\ldots,L$, $\sum_{l=1}^L \Omega_{lj}b_l \leq s_j$, $j = 1,\ldots,J$, $\sum_{j=1}^J \Omega_{lj}b_l\sigma_j \leq \pi_l\left(\sum_{j=1}^J \Omega_{lj}b_l\right)$, $l = 1,\ldots,L$, and $\sum_{l=1}^L \Omega_{lj}b_l\pi_l \geq \sigma_j\left(\sum_{l=1}^L \Omega_{lj}b_l\right)$, $j = 1,\ldots,J$. Then, we can obtain the Lagrangian for (14), given by:

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\Omega}, \mathbf{Z}, \mathbf{x}, \mathbf{y}, \boldsymbol{\mu}, \boldsymbol{\nu}) &= \omega\rho\sum_{j=1}^J\sum_{l=1}^L \Omega_{lj}b_l \\
&+ (1-\omega)\sum_{l=1}^L\sum_{j=1}^J(\Omega_{lj}b_l\pi_l - \Omega_{lj}b_l\sigma_j) + \sum_{l=1}^L\sum_{j=1}^J Z_{lj}\Omega_{lj} \\
&- \sum_{l=1}^L\sum_{j=1}^J x_l(\Omega_{lj}b_l - b_l) - \sum_{l=1}^L\sum_{j=1}^J y_j(\Omega_{lj}b_l - s_j) \\
&- \sum_{l=1}^L\sum_{j=1}^J \mu_l(\Omega_{lj}b_l\sigma_j - \Omega_{lj}b_l\pi_l) \\
&- \sum_{l=1}^L\sum_{j=1}^J \nu_j(\Omega_{lj}b_l\sigma_j - \Omega_{lj}b_l\pi_l).
\end{aligned} \tag{20}$$

Taking first-order derivative of (20) with respect to Ω_{lj} and setting it to zero, we have the following equation at optimality:

$$Z_{lj}^\star = (x_l^\star + y_j^\star - \omega\rho)b_l + \left(\mu_l^\star + \nu_j^\star + (1-\omega)\right)(\sigma_j - \pi_l)b_l. \tag{21}$$

If the lth buyer is not matched with any seller, we have $\sum_{j=1}^J \Omega_{lj}b_l = 0$. By using the complementary slackness at optimality, we have $\Omega_{lj}^\star Z_{lj}^\star = 0$ and $\Omega_{lj}^\star = 0$, $j = 1,\ldots,J \Rightarrow Z_{lj}^\star > 0$, $j = 1,\ldots,J$. We can also derive from $x_l^\star(b_l - \sum_{j=1}^J \Omega_{lj}^\star b_l) = 0$ and $\mu_l^\star \sum_{j=1}^J(\Omega_{lj}^\star b_l\pi_l - \Omega_{lj}^\star b_l\sigma_j) = 0$ that $x_l^\star = 0$ and $\mu_l^\star > 0$. For the mth buyer where $\pi_m < \pi_l$, the price constraint $\sum_{j=1}^J \Omega_{mj}^\star b_m\sigma_j \leq \pi_m\left(\sum_{j=1}^J \Omega_{mj}^\star b_m\right)$ is tighter than the price constraint for the lth buyer so we have $\mu_m^\star > \mu_l^\star$. Since $x_m^\star \geq x_l^\star = 0$ always holds for all the dual variables, we can conclude from the above derivation that $Z_{mj}^\star/b_m \geq Z_{lj}^\star/b_l > 0$, $j = 1,\ldots,J$ (cf. (21)). Hence, we have $Z_{mj}^\star > 0$, $j = 1,\ldots,J$, i.e., the mth buyer, whose purchasing price π_m is lower than π_l, is also unmatched.

Similar proof can be applied to the second bullet point for the sellers in Proposition 5 ■

L. Proof of Proposition 6

Proof: Suppose we have the best matching result by solving (14), which means all the constraints (10)-(13) are tight, then we have $\sum_{j=1}^{J} s_j^\star(\sigma_j) = \sum_{l=1}^{L} b_l^\star(\pi_l) \leq \sum_{l=1}^{L} b_l^\star(\rho)$ and no revenue from the buyer/seller price difference. If revenue of the secondary market is higher than the revenue of the primary market, we have $\rho \sum_{j=1}^{J} s_j^\star(\sigma_j) \;\geq\; p \sum_{l=1}^{L} b_l^\star(p)$. Then, $\rho \sum_{l=1}^{L} b_l^\star(\rho) \geq p \sum_{l=1}^{L} b_l^\star(p)$ implies (16). ■

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